2020

MATHEMATICS — HONOURS

Paper : CC-11

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative with proper justification (wherever applicable) : 2×10

(a) In a game of bridge, the entire deck of 52 cards is dealt out to 4 people. The probability that one of the players receive all 13 spades is

(i) $1/\binom{52}{13}$ (ii) $4/\binom{52}{13}$ (iii) $4\times\binom{52}{13}$ (iv) None of these.

(b) The probability mass function of a random variable X is given by $p(i)=c\dfrac{\lambda^i}{i!}$, $i=0, 1,...$ and c is a constant. Then $P(X=2)$ is given by,

(i) $1-\lambda e^{-\lambda}$ (ii) $\lambda\dfrac{e^{\lambda}}{2}$ (iii) $\lambda^2\dfrac{e^{-\lambda}}{2}$ (iv) None of these.

(c) Let X denote a random variable which takes any of the values $-1, 0$ and 1 with respective probabilities $P(X=-1)=0{\cdot}2$, $P(X=0)=0{\cdot}5$, $P(X=1)=0{\cdot}3$. Then the expectation $E(X^2)$ equals

(i) 0·5 (ii) 0·3 (iii) 0·2 (iv) None of these.

(d) The value of K for which

$$f(x,y)=K(1-x^2-y^2),\quad 0<x^2+y^2<1$$
$$=0 \qquad \text{otherwise}$$

is a probability density function of a two dimensional random variable (X, Y) is given by

(i) $-\dfrac{1}{\pi}$ (ii) $\dfrac{1}{\pi}$ (iii) $\dfrac{2}{\pi}$ (iv) $-\dfrac{2}{\pi}$.

(e) The moment generating function of Binomial distribution with parameters n and p is given by

(i) $(p+qe^t)^n$ (ii) $(pq+e^t)^n$ (iii) $(q+pe^t)^n$ (iv) $(1+pqe^t)^n$.

(f) Suppose X_i, $i = 1, 2, ..., n$ are independently and identically distributed with mean μ and variance σ^2.

Let $S^2 = \frac{1}{n-1}\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2$. Then $E(S^2)$ equals

(i) σ^2 (ii) $(n-1)\sigma^2$ (iii) $\frac{\sigma^2}{n}$ (iv) None of these.

(g) Two random variables X, Y have the least square regression lines with equations $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Then correlation coefficient between them is

(i) $\frac{2}{3}$ (ii) $\frac{1}{2}$ (iii) $-\frac{1}{2}$ (iv) none of these.

(h) Let α be a population parameter which is estimated with two statistics $\boldsymbol{A}$ and $\boldsymbol{B}$ and $P(\boldsymbol{A} < \alpha < \boldsymbol{B}) = \beta$. Then the confidence coefficient for the interval estimate of α with $\boldsymbol{A}$ and $\boldsymbol{B}$ is

(i) β (ii) $\beta - 1$ (iii) $1 - \beta$ (iv) None of these.

(i) Let W be the critical region for the test of a hypothesis H_0 against an alternative hypothesis H_1. If x represents a sample of size n drawn from the population, then $P\left(x \in \bar{W} \mid H_1 \; true\right)$ is the

(i) Probability of Type I error (ii) Probability of Type II error

(iii) Power of the test (iv) None of these.

(j) Which of the following distributions is used to test the equality of standard deviations of two normal populations?

(i) z-distribution (ii) t-distribution (iii) χ^2-distribution (iv) F-distribution.

Unit - 1

Answer ***any two*** questions.

2. Define a σ-algebra (σ-field). Give the definition of probability as a set function. Distinguish between sample space and probability space. 1+2+2

3. A box contains 15 white and 5 black balls and another box contains 7 white and 8 black balls. Two balls are transferred from the first box to the second box. Then one ball is taken from the second box. What is the probability that it is a white ball? What is the probability that no white ball was transferred from the first box to the second box if the ball taken from the second box is found to be white? 3+2

4. Find the characteristic function of the normal distribution $N(0, 1)$. 5

Unit - 2

Answer ***any two*** questions.

5. The joint p.d.f of two variables X, Y is given by

$$f(x,y) = \frac{6-x-y}{8} \quad 0<x<2, \quad 2<y<4$$
$$= 0 \qquad \text{elsewhere}$$

Find $P(X+Y<3)$ and $P(X<1 \mid Y=3)$. 5

6. Suppose (X, Y) is a two-dimensional random variable. Show that $\{E(XY)\}^2 \le E(X^2)\,E(Y^2)$. Deduce, that $-1 \le \rho(X, Y) \le 1$ where ρ is the correlation coefficient between X and Y. 2+3

7. If $f(x, y) = x + y$ $(0 < x < 1,\ 0 < y < 1)$ is the joint density function of variables X and Y, find the distribution of $X + Y$. 5

Unit - 3

Answer ***any one*** question.

8. If X follows a normal distribution with mean m and standard deviation σ, determine $P\left\{|X-m| \ge \frac{3}{2}\sigma\right\}$.

Estimate this probability with the help of Tchebycheff's inequality. [Given $\varphi(1{\cdot}5) = 0{\cdot}9332$] 4+1

9. Show that central limit theorem for equal component implies law of large numbers for equal components. 5

Unit - 4

Answer ***any two*** questions.

10. Let $(x_1, x_2, ..., x_n)$ be any random sample of size n, drawn from a normal distribution $N(0, 1)$. Find the sampling distribution of the sample mean. 5

11. When is an estimator called unbiased and consistent? Construct a minimum variance unbiased estimator out of 5 different unbiased estimator of a parameter θ. 2+3

12. Find the maximum likelihood estimator for parameter p of a binomial (N, P) population. 5

13. Find a 95% confidence interval for the mean of a normal distribution with population variance 9 for the sample 2·3, – 0·2, – 0·4, – 0·9. Given that $\frac{1}{\sqrt{2\pi}}\int_{1\cdot96}^{\infty} e^{-x^2/2}\,dx = 0{\cdot}025$. 5

Please Turn Over

Unit - 5

Answer ***any two*** questions.

14. Explain the following terms with reference to testing of statistical hypothesis : 1+2+2

(a) Null hypothesis

(b) Critical region

(c) Type 1 and Type 2 error.

15. State Neyman–Pearson Lemma and relate it with the idea of critical region. 3+2

16. Fit a parabolic curve of the type $y = a + bx^2$ to the following data using least square method. 5

x	1·0	1·5	2	2·5	3
y	1·1	1·3	1·6	2	2·4

17. For a normal (μ, σ) population, construct a test for the null hypothesis $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1 (> \sigma_0)$ on the basis of a sample from the population where μ is unknown. 5