2020

MATHEMATICS — HONOURS

Paper : DSE-B-2

(Boolean Algebra and Automata Theory)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. All are multiple choice questions with single correct option. Students are required to opt the correct option and justify the correct option in their own words as far as practicable. One mark for correct option and one mark for justification. There is no negative marking. (1+1)×10

(a) If every two elements of a poset are comparable then the poset is

(i) semi lattice (ii) semi group

(iii) totally ordered set (iv) complete lattice.

(b) Finite Automata accepts language that is generated by

(i) Type 0 Grammar (ii) Type 1 Grammar

(iii) Type 2 Grammar (iv) Type 3 Grammar.

(c) A grammar is said to be ambiguous if it produces

(i) more than one derivation (ii) more than one left most derivation

(iii) more than one right most derivation (iv) All of these.

(d) The following CFG

$S \rightarrow aS \mid bS \mid a \mid b$, is equivalent to the regular expression

(i) (a+b)* (ii) (a+b)(a+b)*

(iii) (a+b)*(a+b) (iv) All of these.

(e) Let $\Sigma = \{a, b\}$. Then a regular expression $(a+b)^2$ corresponds to the language

(i) $\{aa, bb\}$ (ii) $\{ab, ba\}$

(iii) $\{aa, ab, ba, bb\}$ (iv) None of these.

(f) The complement of the expression $(A'B + CD')$ is

(i) $(A' + B)(C' + D)$ (ii) $(A + B')(C' + D)$

(iii) $(A'' + B)(C'' + D)$ (iv) $(A + B')(C + D')$.

(g) A problem for which there exists a Turing machine which halts on every input is called

(i) Turing decidable (ii) Turing recognizable

(iii) Both (i) and (ii) (iv) None of these.

(h) Which of the following is/are the universal logic gates?

(i) OR and NOR (ii) AND

(iii) NAND and NOR (iv) NOT.

(i) Let w be a string and fragmented by three variable x, y and z as per pumping lemma. What does these variable represent?

(i) String count (ii) String

(iii) Substring (iv) None of these.

(j) Which of the following machine can accept even palindrome over $\{a, b\}$?

(i) Push down Automata (ii) Turing machine

(iii) NDFA (iv) DFA.

Unit - 1 (5 Marks)

Answer ***any one*** question.

2. Let $n \in N$, where N is the set of all positive integers and S be the set of all divisors of n. In S, define a relation '$\leq$' by $a \leq b$ iff a is a divisor of b for a, $b \in S$. Is $(S, \leq)$ lattice? Justify. 5

3. What is lattice homomorphism? Give an example to show that the order relations are preserved under lattice homomorphism. 2+3

Unit - 2 (10 Marks)

Answer ***any two*** questions.

4. Let $D(40)$ denote the set of all positive divisors of 40. Consider the lattice $(D(40), \leq)$ where $\leq$ denotes the divisibility relation. Find $4 \wedge (8 \vee 10)$ and $(2 \vee (2 \wedge 8)) \vee 20$. 5

5. Use K-map method to find the minimized sum-of-product expression of the following Boolean expression:

$$x\,y\,z + x\,y\,z' + x\,y'\,z' + x'\,y\,z + x'\,y\,z'. \qquad 5$$

6. (a) Define functionally complete operations.

(b) Realize the following switching function using NAND gates only :

$$f(A, B, C, D) = A \cdot C + B \cdot D \qquad 2+3$$

7. Prove that any chain is a distributive lattice. 5

Unit - 3 (10 Marks)

Answer ***any two*** questions.

8. Construct an NFA that accepts the following regular expression : 0*(010)*(00+11). Then convert the NFA into equivalent DFA. 5

9. (a) Suppose that L_1 and L_2 are two languages (over the same alphabet) given to you such that both L_1 and L_1L_2 are regular. Prove or disprove: L_2 must be regular too.

(b) Using the pumping lemma, prove that the language $L_3 = \left\{a^i\, b^j \mid i, j > 0, \text{and} \mid i - j \mid \text{is a prime}\right\}$ is not regular. (Note that 1 is not treated as a prime.) 3+2

10. Construct a Mealy machine that accepts (00+11)*0100 and produce output '*y*' when current input matches previous input and otherwise emits '*n*'. The machine uses its state to remember the last input symbol read. 5

11. (a) Is there any difference between a finite automaton and a finite state machine? Explain.

(b) Let $M = (\Sigma, Q, s_0, \delta, F)$ denote a deterministic finite automaton, where $\Sigma = \{a, b\}, Q = \{s_0, s_1, s_2\}$, $F = \{s_0, s_2\}$ and δ is defined by the following table :

δ	s_0	s_1	s_2
a	s_1	s_2	s_2
b	s_0	s_0	s_1

Draw the state diagram that represents the given automaton M. Hence, determine which one of the following words is accepted by M stating the reason :

aba, bbaaa 1+(2+2)

Unit - 4 (10 Marks)

Answer ***any two*** questions.

12. Construct a Context-Free-Grammar (CFG) with no useless symbol and no €-production from the given grammar:- $G = (V, \Sigma, R, S)$, where $V = \{S, A, a, b\}$, $\Sigma = \{a, b\}$, $R = \{S \to aS, S \to bAB, A \to aA\ AB, B \to A, A \to bS, S \to \varepsilon\}$, $L(G) = \{w \in \{a, b\}^* : w \text{ contains an even number of } b\}$. 5

13. $L = \left\{a^{n1} b^{n2} \ldots\ldots\ldots\ldots a^{nk} b^{nk} \mid k \geq 0\right\}$. Construct a CFG for this language. Prove using Pumping Lemma, whether $\left\{a^M b^M c^N\right\}$ is Context-Free Language or not for $M \geq 1$ and $N \geq M$. 5

14. Convert the grammar with following production rules to Chomsky Normal Form (CNF) :

$$P = \{S \to ASB \mid \Lambda, A \to aAS \mid a, B \to SbS \mid A \mid bb\}\cdot$$ 5

15. Construct a push down automaton which reads the same language as the grammar $\Gamma = (N, \Sigma, S, P)$ defined by $N = \{S, A, B\}$, $\Sigma = \{a, b, c\}$ and set of productions P given by $S \to aA$, $A \to aAB$, $A \to a, B \to b\ \ B \to \Lambda$. 5

Unit - 5 (5 Marks)

Answer ***any one*** question.

16. Construct a *TM* for the language $L = \{0^n 1^n 2^n\}$ where $n \geq 1$. 5

17. Prove that every language accepted by a multitape Turing machine is RE (recursively enumerable). 5

Unit - 6 (5 Marks)

Answer ***any one*** question.

18. Assume that Turing machines are encoded by strings over some alphabet Σ, and that $\# \notin \Sigma$. Consider the following language over the alphabet $\Sigma \cup \{\#\} : L_6 = \{M_1 \# M_2 \# M_3 \mid M_1, M_2, M_3$ are Turing machines with $L(M_1) \cap L(M_2) = L(M_3)\}$. Prove that L_6 is not recursively enumerable. (Note : You must supply a complete reduction proof. No intuitive justification will be given any credit.). 5

19. (a) Does the *PCP* with the following two lists have solution?

$$X = \{100, 0, 1\} \text{ and } Y = \{1, 100, 00\}$$

Justify your answer.

(b) Prove that it is undecidable whether an arbitrary *CFG* (Context Free Grammar) is ambiguous. 2+3